

paper crown



plastic dinosaur



corny joke

I did not work.



MTC
IDEA!

fake mustache





Good Things Come in Unexpected Packages

A Festive Christmas Tradition Leads to Adventures in Problem Solving

Sometimes, a great problem for an MTC session presents itself when you least expect it. In December 2015, on a family visit to England, I sat down to enjoy a longstanding U.K. holiday tradition of opening Christmas crackers. Rather than food, these brightly colored “crackers” are short cardboard tubes wrapped in paper. When pulled apart by two people, with one person holding each end, the friction creates a small explosive “pop.” The crackers contain a variety of small gifts such as tissue paper crowns, novelty toys, and jokes.

When my son opened his Christmas cracker, he discovered “The Mystery Calculator,” a little two-player card game. Of course, I was intrigued and offered to exchange my fake mustache for his game. Little did I know where this gem of a gift would take me.

By Nathan Borchelt



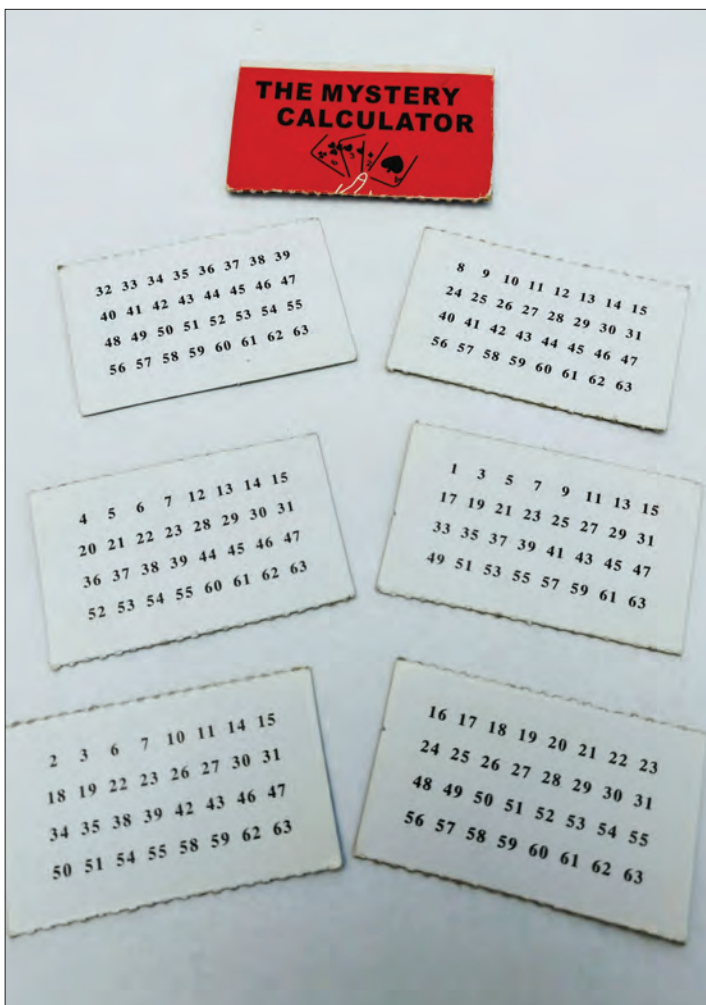
Exploring the Mystery

The Mystery Calculator turned out to be a set of six cards each containing a subset of integers between 1 and 63, as shown at left. The game begins with Player 2 secretly selecting any number visible on any of the cards. Player 1 holds up the cards one at a time, asking Player 2 whether her number appears on the card. From these responses, Player 1 quickly determines which number Player 2 selected. I enjoyed playing the game with members of my family, but obviously, I could not wait to share it with my friends in the Smoky Mountain Math Teachers' Circle. At the session, teachers paired up and each pair received a Christmas cracker. We spent a few minutes enjoying the fun gifts inside and wearing our tissue paper crowns. Then one person in each pair was handed an envelope containing large versions of the mystery calculator cards on pieces of card stock, along with the instructions shown at right.

Each set of partners played the game about five times without Player 1 giving the secret away. After that, we stopped and began our discussion of the game.

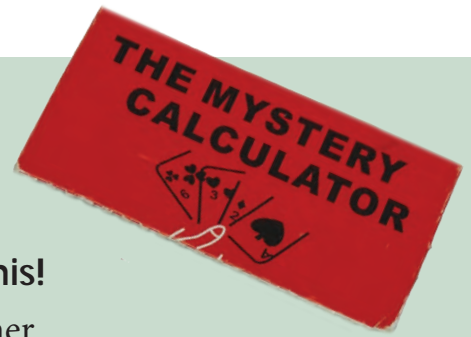
Of course, the natural question is "How does this trick work?" The numbers on the cards seem to be randomly selected numbers between 1 and 63. In fact, the card on which a given number appears depends on its binary representation. As the discussion progressed, most people referred to the cards according to the number in the upper left corner (the 2-card, the 4-card, etc.). It did not take long to recognize these values as powers of 2.

Every integer found on these cards can be broken down as a sum of powers of two. Consider writing them using their 6-digit binary representations. For example, 34 can be written as $2^5 + 2^1 = 32 + 2$. Therefore, the 6-digit binary representation of the number 34 is 100010. The first and fifth digits are 1, which is why the number 34 shows up on both the 2-card and the 32-card.



Top, a stack of unopened Christmas crackers. Above, the contents of the "Mystery Calculator" toy.

Mystery Calculator Instructions



Do the following, but do not let your partner read this!

1. Lay all six cards face up on the table in front of your partner.
2. Ask your partner to secretly select any number they see appearing on any one of the cards.
3. Now ask your partner to look at each card one by one.
4. If they see their number appearing on the card, they should leave the card face up.
5. If their number does not appear on a card, they should flip it over to face down.

Do the magic without sharing your trick with your partner!

Add together the numbers in the upper left corner of the cards that remain face up. Announce this sum as the number that they chose. Aren't you amazing?

For any card that is chosen, every single number on that card will have a 6-digit binary representation with one thing in common. For example, any number that appears on the 8-card has a 1 in the third digit of its 6-digit binary representation (e.g., $29 = 011101$, $46 = 101110$, etc.).

Similarly, any number that appears on the 16-card has a 1 as the second digit in its 6-digit binary representation (e.g., $24 = 011000$, $61 = 111101$, etc.) Exploring these patterns further helps explain why the mystery calculator trick works.

Extensions and Reflection

After cracking the mystery of the Mystery Calculator, participants began to pose and explore some other great questions: What other types of Mystery Calculators can be constructed? Could a different number base be used to do so? What would those cards look like? How many cards would be in the set?

We ran out of time discussing these questions, but that is what made it such a great MTC session.

This is a low-setup, high-fun activity! I prepared plenty of materials so that everyone could leave with their own sets of instructions and a mystery calculator. Several teachers requested additional copies of these simple materials so that they could repeat the activity with their students and colleagues. There has been only one disappointment: I have not yet had anyone else discover a Mystery Calculator inside one of their Christmas crackers. Still, watching teachers wearing tissue crowns and fake mustaches while doing mathematics makes everyone happy. Give this one a try at your next MTC holiday party! 📧

Nathan Borchelt, a co-founder of the Smoky Mountain MTC and the North Carolina Network of MTCs, is an Associate Professor of Mathematics at Western Carolina University.

Resources

For links to this article's resources and more, visit us at www.mathteacherscircle.org/newsletter.



1	3	5	7	9	11	13	15
17	19	21	23	25	27	29	31
33	35	37	39	41	43	45	47
49	51	53	55	57	59	61	63

8	9	10	11	12	13	14	15
24	25	26	27	28	29	30	31
40	41	42	43	44	45	46	47
56	57	58	59	60	61	62	63

32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

16	17	18	19	20	21	22	23
24	25	26	27	28	29	30	31
48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63

2	3	6	7	10	11	14	15
18	19	22	23	26	27	30	31
34	35	38	39	42	43	46	47
50	51	54	55	58	59	62	63

4	5	6	7	12	13	14	15
20	21	22	23	28	29	30	31
36	37	38	39	44	45	46	47
52	53	54	55	60	61	62	63

DON'T SHARE THE FOLLOWING WITH YOUR PARTNER!!

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- 2) Ask him or her to secretly select any number they see appearing on any one of the cards.
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- 5) If their number does not appear on a card, they should flip it over (face down).

THE MYSTERY!!!

Add together the numbers found in the left-hand corner of the cards which remain face up and announce this sum as the number that they chose. Aren't you Amazing????

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