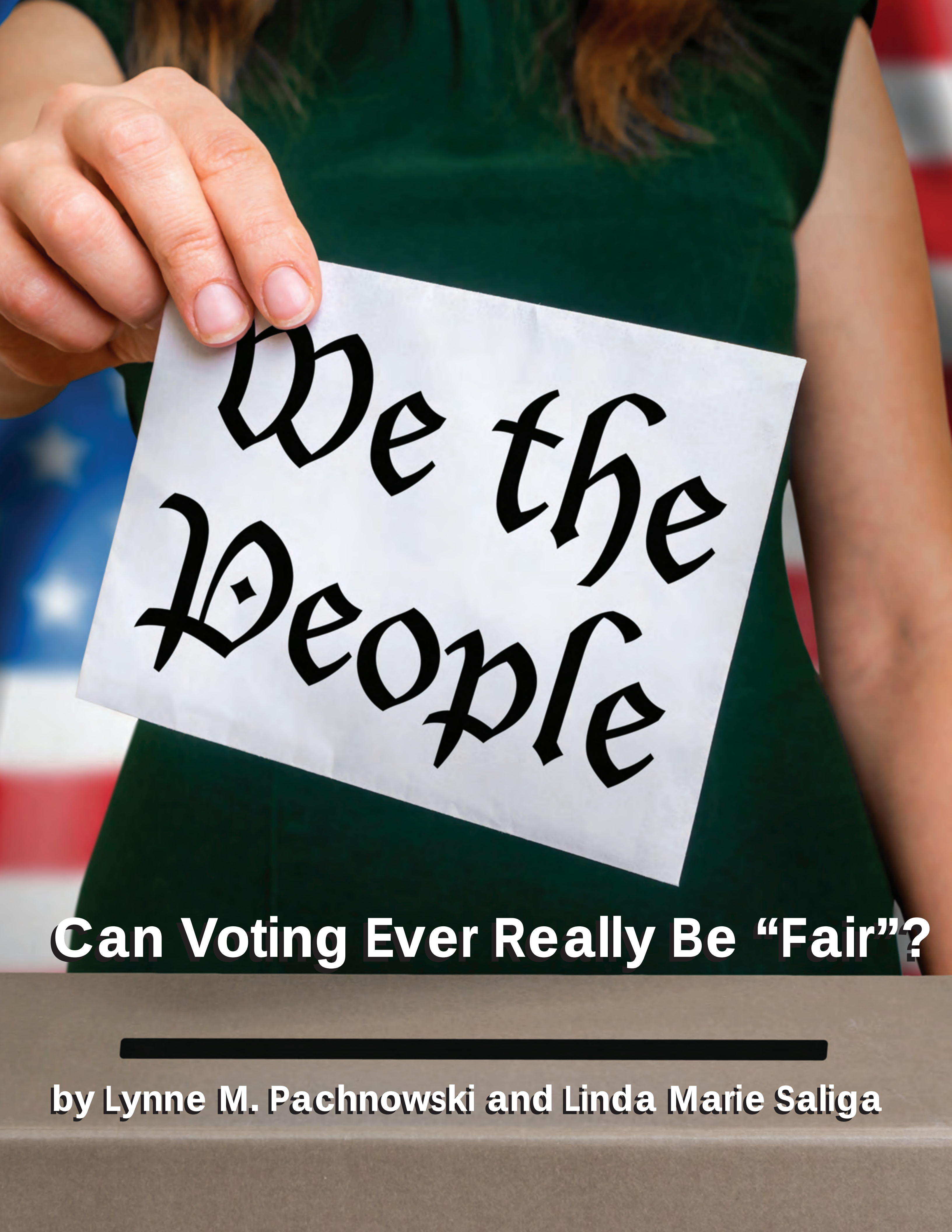

Can Voting Ever Really Be “Fair”?

by Lynne M. Pachnowski and Linda Marie Saliga

What is "fair" when voting? Our Math Teachers' Circle explored voting as a topic before the 2016 Presidential Election. As facilitators, our mathematical objectives were for the participants to apply and analyze several established methods for determining the "voice" of the majority. We wanted them to discover these methods through an inquiry-based experience, by engaging them early in a deep problem and waiting to assign definitions and terms until afterward.

To start, we formed small groups and asked participants to consider the following scenario: One hundred people are asked to rank their preference for the candidate for presidency. The five candidates listed in the table below were the five candidates on Ohio ballots in 2016. The results are as follows. Using all the data given, determine the winner.

TOTAL	37	27	22	11	3
1st	Trump	Clinton	Duncan	Stein	Johnson
2nd	Stein	Stein	Clinton	Johnson	Clinton
3rd	Clinton	Duncan	Stein	Duncan	Duncan
4th	Duncan	Johnson	Trump	Clinton	Stein
5th	Johnson	Trump	Johnson	Trump	Trump

Table 1: The results of our election.

(Yes, it's not realistic to believe that the 100 people created their list in one of only five ways. But for the sake of discussion, we'll use this data to explore.) Note that using **plurality** (the candidate with the most first place votes wins), Trump is the winner. However, some participants objected that this was not the decision of the majority, and it doesn't use all the data.

We asked participants to use all the data provided to determine the winner of the election. Most began by assigning points to each candidate according to where they were ranked on the ballot. That is, one could give 5 points for the candidate in first place, 4 for second place, 3 for third, and so on. Then, the points are multiplied by the number of voters who voted at that level. So, in the first column, 37 voters ranked Trump first, so Trump would get 185 points (37 x 5), then 27 points (27 x 1), etc. The candidate with the highest score wins. In this case, Stein wins. This method is known as the **Borda Count** method.

We then asked, "Is there another way to determine a winner?" Some groups argued that they could consider reassigning all votes to the top two vote-getters, Trump and Clinton, according to everyone's preference regarding these two candidates. In other words, in the first column, since 37 voters ranked Trump over Clinton, all those votes would go to Trump. In the second column, all 27 votes go to Clinton, etc. Using this method, Clinton wins. This method is known as the **Plurality with Runoff** method.

Further prodded, some groups eliminated the candidate with the fewest first-place votes from all the ballots and then recalculated the new plurality totals. For instance, in the table at left, Johnson received only 3 first-place votes. If we eliminate Johnson from the table, essentially conducting a run-off, looking at the first column, those 37 voters would vote the same. However, in the second column, we assume that Trump would now be ranked 4th, and so on. The new table would look as follows:

TOTAL	37	27	22	11	3
1st	Trump	Clinton	Duncan	Stein	Clinton
2nd	Stein	Stein	Clinton	Duncan	Duncan
3rd	Clinton	Duncan	Stein	Clinton	Stein
4th	Duncan	Trump	Trump	Trump	Trump

Table 2: Eliminating the candidate with the fewest first-place votes.

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>
> IN DOING SO, THEY JOIN AN ONGOING DISCUSSION THAT HAS GONE ON HUNDREDS OF YEARS."

Continuing this process, we see that the person with the fewest first-place votes is now Stein, and so we create a new table eliminating her. We repeat that process two more times until we get a winner, in this case Duncan. This method of repeatedly eliminating the candidate with the least first-place votes is the **Hare Rule**.

At this point, we discussed and tried out several other voting methods to see how affected the results. For example, another method, similar to the Hare Rule,

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eliminates the candidate with the most last-place votes. In the original table, Johnson also has the most last place votes. So, the first elimination process would still produce Table 2. However, now looking at Table 2, the candidate with the most last-place votes is Trump, so the next table would eliminate Trump. This method is known as the **Coombs Rule**, and gives the presidency to Clinton.

Another method considers how candidates would fare in a head-to-head battle based on the data. Overall, how did the voters decide between Trump and Clinton? Between Clinton and Stein? In the Trump-Clinton match, 37 percent in the first column would vote for Trump over Clinton, but in all of the remaining columns, voters ranked Clinton over Trump (27 + 22 + 11 + 3 = 63).

vs.	Clinton	Duncan	Johnson	Stein	Trump
Clinton	–	D:33–C:67	J:14–C:86	S:48–C:52	T:37–C:63
Duncan	C:67–D:33	–	J:14–D:86	S:75–D:25	T:37–D:63
Johnson	C:86–J:14	D:86–J:14	–	S:97–J:3	T:59–J:41
Stein	C:52–S:48	D:25–S:75	J:3–S:97	–	T:37–S:63
Trump	C:63–T:37	D:63–T:37	J:41–T:59	S:63–T:37	–
WINS	Clinton: 4 wins	Duncan: 2 wins	Johnson: 0 wins	Stein: 3 wins	Trump: 1 win

Table 3: Comparing candidates using head-to-head matches.

In the head-to-head match, Clinton is the overall winner. This head-to-head method is the **Condorcet Method**.

If the voters simply identify all the candidates they find acceptable, but without ranking (a simple "yes" or "no" method), the candidate that has the most acceptable votes would win. This is **Approval Voting**. When each voter is given a choice to vote for a candidate (+1) or vote against a candidate (-1) and the candidate with the highest sum of the votes cast wins, this is **Negative Voting**. Ballots on which the voters list the candidates in order from top choice to last choice, like the ones used to create our data set, are **Preference Ballots**. Many different methods of determining the winner of an election have been introduced over the years, starting in the Middle Ages.

One participant discussed with great interest a method in which each voter is given a fixed number of points (such as 10) to distribute to the candidates in any way that they please. The candidate with the most points wins, but the voters have been given the option to weight their voting. A voter may choose to give all 10 votes to one candidate. Or, the voter can give 6 votes to one candidate and 4 to another. This method is known as **Cumulative Voting**.

These activities spark a great conversation among participants about what is considered "fair voting." In doing so, they join an ongoing discussion that has gone on hundreds of years. In fact, in 1950, economist, mathematician, and political writer Kenneth Arrow proved that in an election with more than two candidates, it is impossible to develop a preferential voting method that will always be fair.

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