Match or No Match

CSUDH MATH TEACHERS' CIRCLE, FEBRUARY 2018

SHARON LANAGHAN & DR. WAI YAN PONG

The Game

- A two player game that uses a bag with a mixture of blue and red chips
- Each player draws one chips out of the bag
- If the chips match in color, player A wins
- If the chips are different colors, player B wins

Try it!

Using the provided bag, Play 10 rounds of the game, keeping track of how many times player A wins and how many times player B wins

Record your results on the board

The bag should have 3 red chips and 3 blue chips with P(Match)=0.4, P(No Match)=0.6

Try it!

- Does this game seem fair or not? How can you be sure?
- Open the bag and see how many red and blue chips are included. How can this information help determine if the game is fair?
- Can you suggest a different number of red and blue chips that would be fair?

This is a good place to have a conversation about experimental vs theoretical probability and what it means to be fair

The goal of this is to have people come up with ways to calculate the probability. Different groups should share how they calculated the probability to give struggling groups at least a couple of different methods for doing so (tree diagram, using the multiplication rule, etc).

The second question is mostly for groups that finish early. It won't be discussed here, but some groups might make conjectures

Other Games

Your group will be assigned one or more of the games below. For each game, decide if the game is fair and record your answer(s) on the board.

Game	# Blue	# Red	Game	# Blue	# Red
I	1	3	VI	6	6
II	2	5	VII	6	10
III	3	6	VIII	10	15
IV	4	5	IX	12	15
V	5	10	X	15	21

Put labels ‘Fair’ ‘Not Fair’ on the board. Have each group record their given games under ‘Fair’ or ‘Not Fair’.

	# Blue	# Red	P(Match)	P(No Match)
I	1	3	0.5	0.5
II	2	5	0.5238	0.476190476
III	3	6	0.5	0.5
IV	4	5	0.444444444	0.555555556
V	5	10	0.523809524	0.476190476
VI	6	6	0.454545455	0.545454545
VII	6	10	0.5	0.5
VIII	10	15	0.5	0.5
IX	12	15	0.487179487	0.512820513
X	15	21	0.5	0.5

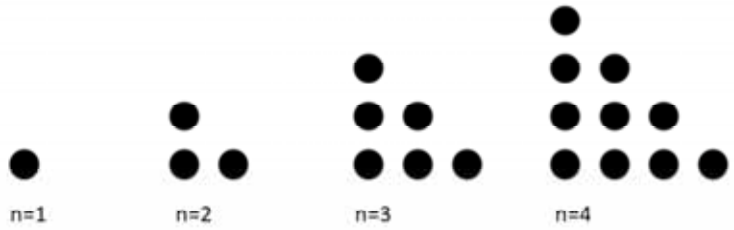
Some Other Problems

For your assigned problem:

1) Make a table for n=1 through n=10
2) Answer the question how many _____ would there be for n=15
3) Write an algebraic equation that models your situation

Some Other Problems

Look at the pattern below. How are the dots increasing? How many dots will there be in the 15^{th} figure?



Some Other Problems

Fifteen mathematicians met up one week.

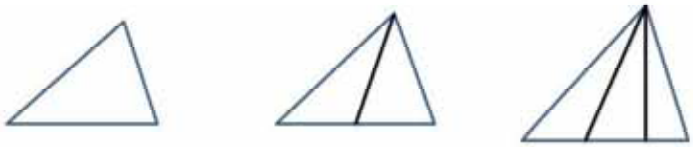
The first mathematician shook hands with all the others.
The second one shook hands with all the others apart from the first one (since they had already shaken hands).
The third one shook hands with all the others apart from the first and the second mathematicians, and so on, until everyone had shaken hands with everyone else.

How many handshakes were there altogether?

https://nrich.maths.org/6708

Some Other Problems

You begin with a triangle and draw a line from one vertex to the opposite side. How many triangles are there? If you repeat this process, how many triangles do you have? How many triangles will you have after you have repeated this process 15 times?



n=1 n=2 n=3

Triangular Numbers

All of the previous problems use triangular numbers.

$$T_n = 1 + 2 + 3 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$$

How does this relate to the match or no match game we started with?

Conjecture

For a game to be fair, the number of red and blue chips must be consecutive triangular numbers.

This is easy (but a little messy) to prove with algebra. Let there be b blue chips and r red chips…

What other mathematical questions can you ask?

1) What happens if you have more colors?
2) What happens if you draw more chips (all chips match for a 'win')?
3) For a given number of chips, how many fair games are there for n colors?
4) What kinds of numbers can be part of a fair game with n colors?

Have teachers come up with questions. (1) and (2) above are likely. (1) can be answered, but is outside the scope of this session, but (2) has not been answered.
For (4), for 3 colors the numbers can be described, but for 4 colors you can build a fair game from any number of chips as one of the colors.

What other mathematical questions can you ask?

A thorough discussion of this problem can be found in the following:

Hui, Chun Yin; Pong, Wai Yan. Diophantine equations of matching games I. Integers 12 (2012), no. 2, 179–195.

https://arxiv.org/abs/1104.3900

Pong, Wai Yan; Stroeker, Roelof J. Diophantine equations of matching games II. Acta Arith. 151 (2012), no. 3, 311–323.

http://www.stroeker.nl/PongStroekerfinal.pdf

Reflection

Choose one of the standards for mathematical practices:

- How was the practice used in this session?
- When/how do you give students in your class opportunities to develop the skills used in the practice?

CCSS Mathematical Practices

OVERARCHING HABITS OF MIND
1. Make sense of problems and persevere in solving them
6. Attend to precision

REASONING AND EXPLAINING
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS
4. Model with mathematics
5. Use appropriate tools strategically

SEEING STRUCTURE AND GENERALIZING
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Match/No Match

Some Other Problems

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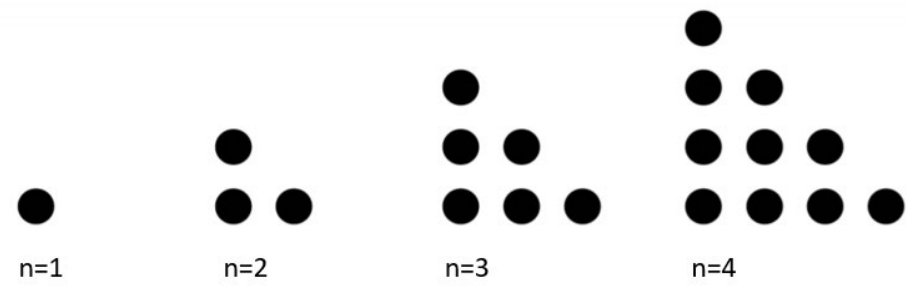

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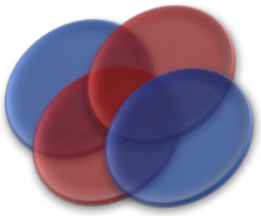
2) How many dots will there be for n=15?
3) Write an algebraic equation that models your situation



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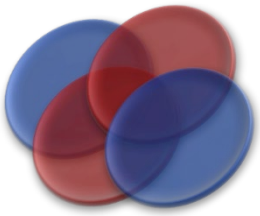
How many handshakes were there altogether?

1) Make a table for two through 10 people
2) Answer the question how many handshakes would there be for 15 people?
3) Write an algebraic equation that models your situation

Match/No Match

Some Other Problems



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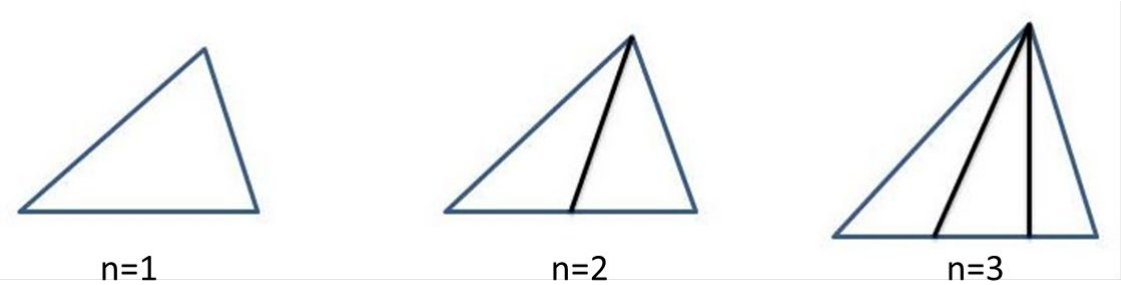

1) Make a table for n=1 to n=10
2) How many triangles will there be for n=10?
3) Write an algebraic equation that models your situation