Supreme Court Handshakes
Sample Models

Model # 1: Circle

Step # 1: Create a circle with dots representing the number of justices.

Step # 2: Starting with a circle, draw lines to each of the other circles presenting shaking hands with each of the other justices one time.
Step # 3: Repeat step # 2 with each of the remaining circles.

Suggestion: It might be helpful to use different colors to represent each of the different justices shaking hands.
## Model # 2: Table

<table>
<thead>
<tr>
<th>Number of Justices</th>
<th>Total Handshakes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
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<tr>
<td>5</td>
<td>10</td>
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<tr>
<td>6</td>
<td>15</td>
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<tr>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>8</td>
<td>28</td>
</tr>
<tr>
<td>9</td>
<td>36</td>
</tr>
<tr>
<td>10</td>
<td>45</td>
</tr>
<tr>
<td>30</td>
<td>435</td>
</tr>
<tr>
<td>100</td>
<td>4950</td>
</tr>
<tr>
<td>( \times )</td>
<td>( \frac{\times (\times - 1)}{2} )</td>
</tr>
</tbody>
</table>
Model # 3: List

Scenario with 9 Justices where each Justice is a number 1 – 9 and the combination of numbers represents the shaking of hands

*Starting with Judge #1, notice on with every next person there is one less handshake because they have already shaken hands.

1 – 2, 1 – 3, 1 – 4, 1 – 5, 1 – 6, 1 – 7, 1 – 8, 1 – 9 [8 shakes]
2 – 3, 2 – 4, 2 – 5, 2 – 6, 2 – 7, 2 – 8, 2 – 9 [7 shakes]
3 – 4, 3 – 5, 3 – 6, 3 – 7, 3 – 8, 3 – 9 [6 shakes]
4 – 5, 4 – 6, 4 – 7, 4 – 8, 4 – 9 [5 shakes]
5 – 6, 5 – 7, 5 – 8, 5 – 9 [4 shakes]
6 – 7, 6 – 8, 6 – 9 [3 shakes]
7 – 8, 7 – 9 [2 shakes]
8 – 9 [1 shake]

*Notice that for 9 Justices, you can determine the total number of handshakes with this model and calculating

\[ 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36 \]
Model # 4: Matrix of handshakes

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>1-2</td>
<td>1-3</td>
<td>1-4</td>
<td>1-5</td>
<td>1-6</td>
<td>1-7</td>
<td>1-8</td>
<td>1-9</td>
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<td>9-8</td>
<td>9-9</td>
</tr>
</tbody>
</table>

- The red cells represent handshakes that can’t occur because a supreme court justice would be shaking their own hand.

- The yellow and blue cells are mirror images of each other. This helps students see why we divide by 2 when we create the general formula for n handshakes.
Model # 5: Cube manipulatives

One cube represents one handshake. The image below, is showing nine justices. The column is each justice and the number of handshakes that justice makes. The total number of cubes is the total number of handshakes of the nine judges at one gathering.

![Image of cube manipulatives](image)

This model can be used to help students develop the equation for $n$ justices.

- Take two copies of one justice and put them together to see what shape they make. [The two identical pieces together will make a rectangle. Total number of handshakes is half of the area of this rectangle.]
One cube represents one handshake. The image below, is showing four justices. The column is each justice and the number of handshakes that justice makes. The total number of cubes is the total number of handshakes of the four judges at one gathering.

This model can be used to help students develop the equation for $n$ justices.

- Take two copies of one justice and put them together to see what shape they make. [The two identical pieces together will make a rectangle. Total number of handshakes is half of the area of this rectangle.]
1. Topic Summary

**Topic:** Identifying Patterns, Developing Algebraic Expressions and Reasoning, Modeling with Mathematics using Multiple Representations

**Summary:** This lesson requires students to use their problem solving skills in order to determine the relationship between the number of Supreme Court justices and handshakes that occur when each person shakes hands exactly once. Students will begin exploring with simpler numbers and work up to creating an algebraic expression to represent the function. This lesson allows for multiple representations by using a table, list, circle diagram, matrix and manipulatives. Depending the pace of the students, this lesson can take up to 1-2 hours to implement in a Math class.

Once students complete this module, they should be able to:
- Identify patterns between two quantities
- Model the relationship using a table, list, circle diagram, manipulatives, matrix
- Develop an algebraic expression based on the situation

**Rationale:** The purpose of these tasks is for students to identify patterns and develop an algebraic expression based on the relationship between two quantities. It is a good introduction to the 8th grade or Algebra Math curriculum using inquiry based instruction.

**Core alignments:**

CCSS.MATH.CONTENT.8.F.A.1
Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.¹

CCSS.MATH.CONTENT.HSF.BF.A.1
Write a function that describes a relationship between two quantities.*

**Other learning goals:**
Students will be able to create an algebraic expression based on a situation.
Students will be able to model a situation using a table, list, circle diagram, matrix, or manipulatives.

**Materials:** Handout, Supplemental worksheets, Cube/Square manipulatives would be helpful, colored pencils
Inquiry-Based Course Module
Supreme Court Welcome

**Prerequisites:** We expect all students to be able to do this task. It would be helpful for students to understand the concept of a variable when developing the algebraic expression.

**Outcomes:** Upon completion, students will be able to identify patterns, model a situation and develop an algebraic expression.
Activity: This is probably where the bulk of the work goes. Describe the activity/activities. Put leading questions that the students might do. Describe how things get introduces and how it works. Basically, tell us what the module is about. Refer to attached worksheets as necessary.

- Present the problem: Every time the Supreme Court justices get together, everyone shakes hands with each other. How many total handshakes will take place at one gathering?

- Review what we know/need to know in order to solve this problem.

- Students begin working to create a model in order to solve the problem.

- Students share their model with others.

- Students try out a new model that someone shared.

- Learn about the history of the number of Supreme Court justices over the years.

- Play out the situation with many different justices and determine the pattern of justices to total handshakes.

- Develop an algebraic expression to represent the number of handshakes for any number of people.

Follow up: Include, if appropriate, follow-up activities for students to do for homework, for enrichment, for practice, or for other purposes.

An interesting sequel could be to determine how long it would take in order to complete the Supreme Court handshake. This is an open-ended question because you can modify it however you would like in order to fit the needs of your students.
3. Instructor Notes

Instructor notes: Include notes to the instructor about how to use this module, things to be mindful of, suggestions, advice, ideas, things that have worked in the past, things that have not worked, etc. These can be very valuable and can be edited along the way.

- Allow students to try to digest and think about the problem in their own time
- It is important to guide their thinking, but allow them to arrive at the ideas by themselves
- Brainstorming together and sharing methods is helpful
- When filling out the know and need to know chart - we know that every person shakes hands exactly once and we need to know how many supreme court justices there are and how many total handshakes there will be
- Students should be trying to create an illustration/diagram/model on their own at first (if they are struggling try to brainstorm and provide examples of different models)
- Teaching them the history should get them to work on determining the total handshakes for different numbers of justices other than 9
- The hope is to get them to see that the algebraic expression is half of the product of n and n-1 where n is the number of people (For instance, if there are 9 people n would be 9 and n-1 would be 8 and half of their product is 36 which will be the total handshakes for 9 people)

Student examples: If you have any examples of (anonymous) student work that might help future instructors, include them. Otherwise, how are students likely to approach these tasks? What will their misunderstandings be? How can we guide students around these misunderstandings?

- Students are likely to forget that a person will not shake their own hand.
- Students are typically inclined to think that you can multiply the number of people by how many different people’s hands they will shake.
- A good way to guide them is by acting out a scenario of classmates shaking hands.

*Please see the model document to see how this problem is worked out using a table, list, circle diagram, matrix and cube manipulatives.
4. MTC dissemination activity

Plan for dissemination: What activities will you do in order to share this module with other teachers at a 2-hour Math Teachers’ Circle meeting? How about a conference presentation at AMTRA or other appropriate venue? What will the participants in a MTC session do? Try to make this as active as possible.

I will present this module in the Math Teachers’ Circle as if I am teaching it in my class. Attendants will be able to get a sense of how this module should look in the classroom and how they should adjust it for their own population. I am also hoping for feedback from attendants in order to make this module better.
5. Notes for MTC dissemination

Facilitator notes for dissemination: What notes do you have for yourself (or anyone else sharing these materials with others) during a 2-hour Math Teachers’ Circle session or other planned conference presentation? These accompany the work outlined in the last section.

My biggest takeaway from the presentation was how I chose to word my instructions to my students. When I introduced the building blocks, I directed everyone to build “the process” of 4 people shaking hands. This caused confusion and ultimately we spoke about “progression” being a better term in order to guide student exploration. With that being said though, it is important to be comfortable watching the struggle and thinking process for inquiry based learning. Many teachers want to interject and guide students directly to the answer. However, it is important we are patient and allow our students to arrive at the answer in their own way so it makes sense to them.